

Solution for exercise 5.4.3 in Pitman

For $\alpha = \beta$ we have the *Gamma*(2, α) distribution. We denote the waiting time in queue i by X_i , and the total waiting time by Z .

Question a) The distribution of the total waiting time Z is found using the density convolution formula page 372 for independent variables.

$$f(t) = \int_0^t \alpha e^{-\alpha u} \beta e^{-\beta(t-u)} du = \alpha \beta e^{-\beta t} \int_0^t e^{u(\beta-\alpha)} du = \frac{\alpha \beta}{\beta - \alpha} (e^{-\alpha t} - e^{-\beta t})$$

Question b)

$$E(Z) = E(X_1) + E(X_2) = \frac{1}{\alpha} + \frac{1}{\beta}$$

See e.g. page 480 for the means $E(X_i)$ for the exponential variables .

Question c) Using the independence of X_1 and X_2 we have

$$Var(Z) = Var(X_1) + Var(X_2) = \sqrt{\frac{1}{\alpha^2} + \frac{1}{\beta^2}}$$

The last equalit follows from e.g. page 480.