IMM - DTU

02405 Probability
2003-10-17
BFN/bfn

Solution for exercise 5.3.9 in Pitman

Question a) We will do all the calculations in inches, noting that 1 foot is 12 inches. We have the distribution of the largest observation from page 316

$$P(X_{(100)} > 76) = 1 - P(X_{(100)} \leq 76) = 1 - \left(\Phi\left(\frac{76-70}{2}\right)\right)^{100} = 1 - 0.9987^{100} = 0.122$$

(Pitman 0.1307??? which is what you would get with $P(X_{(100)} \leq 76) = 0.9986$)

Question b) The average of $\bar{X}$ of the 100 observations is given by $\bar{X} = \frac{1}{100}\sum_{i=1}^{100} X_i$. We have from the boxed result page 363 that $\sum_{i=1}^{100} X_i$ is normally distributed implying that $\bar{X}$ is normally distributed. We find mean and standard deviation of $\bar{X}$ using the Square Root Law page 194 such that

$$P(\bar{X} > 70.5) = 1 - \Phi\left(\frac{70.5-70}{\frac{2}{\sqrt{100}}}\right) = 1 - \Phi(2.5) = 0.0062$$

Question c) Page the Central Limit Theorem (e.g. page 386) can be applied in case b). Limit theorems exists for maximum and minimum of random variables (extreme value distributions). These results depend on the specific form of the distribution of the individual X_i's. One can easily construct counterexamples to disprove the generality of a), like uniformly distributed X_i's.