Solution for exercise 5.3.3 in Pitman

Question a)

$$P(W + X > Y + Z + 1) = P(W + X - Y - Z > 1)$$

The variable $V = (W + X - Y - Z) \in Normal(0, 4)$ Thus

$$P(W+X > Y+Z+1) = P(V > 1) = P\left(\frac{V-0}{2} > \frac{1}{2}\right) = 1 - \Phi\left(\frac{1}{2}\right) = 1 - 0.6915$$

Question b)

$$P(4X + 3Y < Z + W) = P(4X + 3Y - Z - W < 0) = 0.5$$

Question c)

$$E(4X + 3Y - 2Z^2 - W^2 + 8) = 4E(X) + 3E(Y) - 2E(Z^2) - E(W^2) + 8$$

from the standard result: the expectation of a linear expression is the linear expression of the expectations.

$$4E(X) + 3E(Y) - 2E(Z^2) - E(W^2) + 8 = -2 - 1 + 8 = 5$$

since X, Y, Z, W are standard normal variables.

Question d)

$$Var(3Z - 2X + Y + 15) = 9Var(Z) + 4Var(X) + Var(Y) = 14$$

since X, Y, Z are independent and standard normal variables.

$$SD(3Z - 2X + Y + 15) = \sqrt{Var(3Z - 2X + Y + 15)} = \sqrt{14}$$