IMM - DTU

02405 Probability
2004-4-11
BFN/bfn

Solution for exercise 5.3.1 in Pitman

Question a) With R being the distance from the bulls eye to the shot we have (page.360 line b.4)

$$P(R \leq r) = F_R(r) = 1 - e^{-\frac{1}{2}r^2}$$

thus

$$F_R\left(\frac{1}{2}\right) = 1 - e^{-\frac{1}{8}} = 0.1175$$

Question b)

$$P(1 \leq R \leq 2) = F_R(2) - F_R(1) = e^{-\frac{1}{2}} - e^{-2} = 0.4712$$

we get $\frac{1}{4} \cdot 0.4712 = 0.1178$ due to the symmetry.

Question c) We are considering only the second coordinate, which is a standard normal variable. The mean of the absolute value is given page 484.

$$\sqrt{\frac{2}{\pi}}$$

Question d) This is the probability that the absolute value of the first coordinate is less than or equal to $r = 1.1777\ldots$.

$$(2\Phi(r) - 1) \tilde{=} (2\Phi(1.18) - 1) = 0.762$$

Question e) This is the probability that the absolute value of largest of the two coordinates are less than or equal to r.

$$(2\Phi(r) - 1)^2 \tilde{=} (2\Phi(1.18) - 1)^2 = 0.581$$

Question f) Use rotational symmetry and find similarly to e)

$$(2\Phi\left(\frac{r}{\sqrt{2}}\right) - 1)^2 = 0.352$$

Question g)

$$\frac{1}{2}(2\Phi(r) - 1)^2 = 0.29$$