

Solution for exercise 5.2.8 in Pitman

Question a) We find the marginal density of Y by integrating over x (page 349)

$$f_Y(y) = \int_{-y}^y c(y^2 - x^2)e^{-y} dx = c \frac{4}{3} y^3 e^{-y}$$

We recognize this as a gamma density (1) page 286 with $\lambda = 1$ and $r = 4$ thus $c = \frac{1}{8}$

Question b) With $Z = g(Y) = 4Y^3$, $\frac{dg(y)}{dy} = 12y^2$, $Y = \left(\frac{Z}{4}\right)^{\frac{1}{3}}$, using the boxed result page 304 we get

$$f_Z(z) = \frac{y^3}{6} e^{-y} \frac{1}{12y^2} = \frac{\left(\frac{z}{4}\right)^{\frac{1}{3}}}{72} e^{-\left(\frac{z}{4}\right)^{\frac{1}{3}}}$$

Question c) We have $|X| \leq |Y| = Y$. Thus $E(|X|) \leq E(Y) = 4$.