IMM - DTU

02405 Probability
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BFN/bfn

# Solution for exercise 5.2.7 in Pitman

We denote the radius of the circle by $\rho$. The are of the circle is $\pi\rho^2$. If a chosen point is within radius $r$ it has to be within the circle of radius $r$ with area $\pi r^2$. We find the probability as the fraction of these two areas

$$F_R(r) = P(R_1 \leq r) = \frac{r^2}{\rho^2}$$

with density (page 333)

$$f_R(r) = \frac{\mathrm{d}F_R(r)}{\mathrm{d}r} = \frac{2r}{\rho^2}$$

With $R_1$ and $R_2$ indpendent we have the joint density from (2) page 350

$$f(r_1, r_2) = \frac{4r_1 r_2}{\rho^4}$$

We now integrate over the set $r_2 < \frac{r_1}{2}$ (page 349) to get

$$P\left(R_2 \leq \frac{R_1}{2}\right) = \int_0^\rho \int_0^{\frac{r_1}{2}} \frac{4r_1 r_2}{\rho^4} \mathrm{d}r_2 \mathrm{d}r_1 = \frac{1}{2\rho^4} \int_0^\rho r_1^3 \mathrm{d}r_1 = \frac{1}{8}$$