IMM - DTU

02405 Probability
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BFN/bfn

Solution for exercise 5.2.6 in Pitman

Question a) Draw a small figure showing the area of integration. Using page 349 we get

$$P(Y > 2X) = \int_0^{\frac{1}{2}} \int_{2x}^1 90(y-x)^8 \mathrm{d}y\mathrm{d}x = \int_0^{\frac{1}{2}} \left[10(y-x)^9\right]_{y=2x}^{y=1} \mathrm{d}x = \int_0^{\frac{1}{2}} (10(1-x)^9 - 10x^9)\mathrm{d}x$$

$$= \left[-(x-1)^{10} - x^{10}\right]_{x=0}^{x=\frac{1}{2}} = 1 - 2\left(\frac{1}{2}\right)^{10}$$

Question b) The marginal density of X is given by (using page 349)

$$f_X(x) = \int_x^1 90(y-x)^8 \mathrm{d}y = 10(1-x)^9$$

with CDF

$$F_X(x) = \int_0^x 10(1-u)^9 \mathrm{d}u = 1 - (1-x)^{10}$$

The marginal density of Y is given by (using page 349)

$$f_Y(y) = \int_0^y 90(y-x)^8 \mathrm{d}y = 10y^9$$

with CDF

$$F_Y(y) = \int_0^y 10u^9 \mathrm{d}u = y^{10}$$

Question c) Maximum and minimum (see exercise 4.6.3). Also note that the marginal distributions are those of max and min from page 316/317.