IMM - DTU

02405 Probability
2003-11-22
BFN/bfn

Solution for exercise 5.2.11 in Pitman

Question a)

$$E(X+Y) = E(X) + E(Y) = 1.5$$

from the general rule of the expectation of a sum.

Question b)

$$E(XY) = E(X)E(Y) = 0.5$$

by the independe of X and Y.

Question c)

$$E((X-Y)^2) = E(X^2+Y^2-2XY) = E(X^2)+E(Y^2)-2E(XY)$$

$$= (Var(X)+(E(X))^2)+(Var(Y)+(E(Y))^2)-2E(XY) = \frac{1}{12}+\frac{1}{4}+1+1-1 = \frac{4}{3}$$

from the general rule of the expectation of a sum, the computational formula for the variance, and the specific values for the uniform and exponential distributions.

Question d)

$$E\left(X^2e^{2Y}\right) = E(X^2)E\left(e^{2Y}\right)$$

We recall the general formula for $E(g(Y))$ from page 263 or 332

$$E(g(Y)) = \int_y g(y)f(y)\mathrm{d}y$$

where $f(y)$ is the density of Y. Here Y is $exponential(1)$ distributed with density $f(y) = 1\cdot e^{-1\cdot y}$. We get

$$E\left(e^{2Y}\right) = \int_0^\infty e^{2y}1\cdot e^{-y}\mathrm{d}y = \infty$$

thus $E\left(X^2e^{2Y}\right)$ is undefined (∞).