Solution for exercise 5.1.1 in Pitman

Question a) Consider the area of the support for the density to get

$$P(X > 1) = \frac{\frac{5}{2}}{6} = \frac{5}{12} \qquad P(X \le 1) = \frac{7}{12}$$

or integration of

$$\int_0^1 \int_x^4 \frac{1}{6} \mathrm{d}y \mathrm{d}x = \int_0^1 \frac{4-x}{6} \mathrm{d}x = \frac{4 - \frac{1}{2}}{6} = \frac{7}{12}$$

Question b)

$$\int_1^2 \int_x^{x^2} \frac{1}{6} \mathrm{d}y \mathrm{d}x = \frac{1}{6} \int_1^2 (x^2 - x) \mathrm{d}x = \frac{1}{6} \left[\frac{1}{3}x^3 - \frac{1}{2}x^2 \right]_{x=1}^{x=2} = \frac{5}{36}$$

(note that $x^2 < x$ for $0 < x < 1$)