IMM - DTU

02405 Probability
2005-4-7
BFN/bfn

Solution for review exercise 1 (chapter 1) in Pitman

Solution for exercise 4.6.4 in Pitman

Question a)

$$P(Z=1) = P(S<T) = P(S \leq T) = P(T \leq S) = P(Z=0) \Rightarrow P(Z=1) = P(Z=0) = \frac{1}{2}$$

Question b) It is intuitevly tempting to claim that X and Z are independent. This is an example where intuition is correct. However one should be careful and should be able to verify with rigourous arguments.

$$P(X \leq x | Z=1) = \frac{P(X \leq x, Z=1)}{P(Z=1)} = \frac{P(S \leq x, S<T)}{\frac{1}{2}}$$

Now

$$P(X \leq x) = P(S \leq x, S<T) + P(T \leq x, T<S) = 2P(T \leq x, T<S)$$

Inserting we get

$$P(X \leq x | Z=1) = \frac{\frac{P(X \leq x)}{2}}{\frac{1}{2}} = P(X \leq x)$$

A similar argument shows the independence of Y and Z.

Question c) Indpendence between which variable attains the k'th order statistic and the value of the k'th order statistic.