Solution for exercise 4.6.3 in Pitman

Question a)

$$P(U_{(1)} \geq x, U_{(n)} \leq y) = P(x \leq U_1 \leq y, x \leq U_2 \leq y, \ldots x \leq U_n \leq y) = (y - x)^n$$

Question b)

$$P(U_{(1)} \geq x, U_{(n)} > y) = P(U_{(1)} \geq x) - P(U_{(1)} \geq x, U_{(n)} \leq y) = (1 - x)^n - (y - x)^n$$

Question c)

$$P(U_{(1)} \leq x, U_{(n)} \leq y) = P(U_{(n)} \leq y) - P(U_{(1)} \geq x, U_{(n)} \leq y) = y^n - (y - x)^n$$

Question d)

$$1 - (1 - x)^n - y^n + (y - x)^n$$

Question e)

$$\binom{n}{k} x^k (1 - y)^{n-k}$$

Question f)

$$k < x, n - k - 1 > y$$

one in between