IMM - DTU

02405 Probability
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BFN/bfn

Solution for exercise 4.5.8 in Pitman

We let X_i denote the lifetime of the i'th component, and S denote the lifetime of the system.

Question a) We have the maximum of two exponential random variables $S = \max(X_1, X_2)$.

$$P(S \leq t) = P(\max(X_1, X_2) \leq t) = \left(1 - e^{-\lambda_1 t}\right)\left(1 - e^{-\lambda_2 t}\right)$$

from page 316 and example 4 page 317/318. Thus

$$P(S > t) = 1 - \left(1 - e^{-\lambda_1 t}\right)\left(1 - e^{-\lambda_2 t}\right) = e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1+\lambda_2)t}$$

Question b) In this case we have $S = \min(X_1, X_2)$ and we apply the result for the minimum of random variables page 317. The special case of two exponentials is treated in example 3 page 317

$$P(S \leq t) = 1 - e^{-(\lambda_1+\lambda_2)t}$$

Question c) From the system design we deduce $S = \max(\min(X_1, X_2), \min(X_3, X_4))$ such that

$$P(S \leq t) = \left(1 - e^{-(\lambda_1+\lambda_2)t}\right)\left(1 - e^{-(\lambda_3+\lambda_4)t}\right)$$

Question d) Here $S = \min(\max(X_1, X_2), X_3)$ such that

$$P(S \leq t) = 1 - \left(1 - \left(1 - e^{-\lambda_1 t}\right)\left(1 - e^{-\lambda_2 t}\right)\right)e^{-\lambda_3 t} = 1 - e^{-(\lambda_1+\lambda_3)t} - e^{-(\lambda_2+\lambda_3)t} + e^{-(\lambda_1+\lambda_2+\lambda_3)t}$$