Solution for exercise 4.5.6 in Pitman

Question a) From the definition of the cumulative distribution function page 313 we get

$$P\left(X \geq \frac{1}{2}\right) = 1 - P\left(X < \frac{1}{2}\right) = 1 - P\left(X \leq \frac{1}{2}\right)$$

where the last equality is true for continuous distributions.

$$P\left(X \geq \frac{1}{2}\right) = 1 - P\left(X \leq \frac{1}{2}\right) = 1 - F\left(\frac{1}{2}\right) = \frac{7}{8}$$

Question b) The density is the first derivative of the CDF for a continuous distribution (page 313), thus

$$f(x) = \frac{\mathrm{d}F(x)}{\mathrm{d}x} = 3x^2$$

Question c) We calculate the mean from the definition page 261

$$E(X) = \int_0^1 x f(x)\mathrm{d}x = \int_0^1 x \cdot 3x^2 \mathrm{d}x = \frac{3}{4}$$

Question d) The variables Y_1, Y_2, and Y_3 are all uniformly distributed with CDF $F_Y(y) = y$ (see eg. page 315). The discussion on the distribution of maximum of n indpendent random variables page 316 tells us that $Z = \max(Y_1, Y_2, Y_3)$ with CDF $F_Z(z)$

$$F_Z(z) = P(Z \leq z) = P(\max(Y_1, Y_2, Y_3) \leq z) = P(Y_1 \leq z, Y_2 \leq z, Y_3 \leq z)$$
$$= P(Y_1 \leq z)P(Y_2 \leq z)P(Y_3 \leq z) = z^3$$