

Solution for exercise 4.5.4 in Pitman

The operations considered are shifting (addition of b) and scaling (multiplication by a). We introduce $Y = aX + b$. The distribution $F_Y(y)$ of Y is given by

$$F_Y(y) = P(Y \leq y) = P(aX + b \leq y) = P(aX \leq y - b)$$

For $a > 0$ we get

$$F_Y(y) = P\left(X \leq \frac{y - b}{a}\right) = F\left(\frac{y - b}{a}\right)$$

For $a < 0$ we get

$$F_Y(y) = P\left(X \geq \frac{y - b}{a}\right) = 1 - P\left(X \leq \frac{y - b}{a}\right) = 1 - F\left(\frac{y - b}{a}\right)$$