IMM - DTU

02405 Probability
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BFN/bfn

Solution for exercise 4.4.6 in Pitman

We have

$$\tan(\Phi) = y$$

and use the change of variable result page 304 to get

$$\frac{\mathrm{d}\tan(\Phi)}{\mathrm{d}\Phi} = 1 + \tan(\Phi)^2 = 1 + y^2$$

Now inserting into the formula page 304 we get

$$f_Y(y) = \frac{1}{\pi}\frac{1}{1+y^2}, -\infty < y < \infty$$

The function is symmetric ($f_Y(y) = f_Y(-y)$) since $(-y)^2 = y^2$, but

$$\int_0^a y \cdot \frac{1}{\pi}\frac{1}{1+y^2}\mathrm{d}y = \frac{1}{2\pi}\ln(1+a^2) \rightarrow \infty \text{ for } a \rightarrow \infty$$

The integral $\int_{-\infty}^{\infty} y f_Y(y)\mathrm{d}y$ has to converge absolutely for $E(Y)$ to exist, i.e. $E(Y)$ exists if and only if $E(|Y|)$ exists (e.g. page 263 bottom).