IMM - DTU

02405 Probability
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BFN/bfn

Solution for exercise 4.4.3 in Pitman

First we introduce $Y = g(U) = U^2$ and note that $g()$ is strictly increasing on $]0, 1[$. We then apply the formula in the box on page 304. In our case we have

$$f_X(x) = 1 \text{ for } 0 < x < 1, \qquad y = g(x) = x^2, \qquad x = \sqrt{y}, \qquad \frac{\mathrm{d}y}{\mathrm{d}x} = 2x = 2\sqrt{y}$$

Inserting in the formula

$$f_Y(y) = \frac{1}{2\sqrt{y}} \qquad 0 < y < 1$$

Alternative solution using cumulative distribution - section 4.5

$$F_{U^2}(y) = P(U^2 \leq y) = P(U \leq \sqrt{y}) = \sqrt{y}$$

The last equality follows from the cumulative distribution function (CDF) of a Uniformly distributed random variable (page 487). The density is derived from the CDF by differentation (page 313) and

$$f_{U^2}(y) = \frac{\mathrm{d}F_{U^2}(y)}{\mathrm{d}y} = \frac{1}{2\sqrt{y}}, 0 < y < 1$$