IMM - DTU

02405 Probability
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Solution for exercise 4.4.1 in Pitman

We apply boxed results page 304. First we introduce $Y = g(X) = cX$ and note that $g()$ is strictly increasing. We have

$$f_X(x) = \lambda e^{-\lambda x} \text{ for } 0 < x, \qquad y = g(x) = c \cdot x, \qquad x = \frac{y}{c}, \qquad \frac{\mathrm{d}y}{\mathrm{d}x} = c$$

Inserting in the formula

$$f_Y(y) = \frac{\lambda e^{-\lambda \frac{y}{c}}}{c} = \frac{\lambda}{c} e^{-\frac{\lambda}{c} y} \qquad 0 < y < 1$$

such that Y follows an exponential distribution with parameter(intensity) $\frac{\lambda}{c}$.

Alternative solution using cumulative distribution - section 4.5

We define a new random variable $Y = cX$. The distribution of Y

$$P(Y \leq y) = P(cX \leq y) = P\left(X \leq \frac{y}{c}\right) = 1 - e^{-\lambda \frac{y}{c}} = 1 - e^{-\frac{\lambda}{c} y}$$