IMM - DTU

02405 Probability
2003-11-12
BFN/bfn

Solution for exercise 4.3.6 in Pitman

The hazard rate is given by

$$\lambda(t) = \begin{cases} 0.05 & t \leq 10 \\ 0.1 & t > 10 \end{cases}$$

Question a) Using the relation (7) page 297 we get

$$G(t) = e^{-(10 \cdot 0.05 + 5 \cdot 0.1)} = e^{-1}$$

Question b)

$$G(t) = \begin{cases} e^{-0.05t} & t \leq 10 \\ e^{-0.5} e^{-0.1(t-10)} & t > 10 \end{cases}$$

Question c) Using (5) page 297 we get

$$f(t) = \begin{cases} 0.05 e^{-0.05t} & t \leq 10 \\ 0.1 e^{-0.5} e^{-0.1(t-10)} & t > 10 \end{cases}$$

Question d) We calculate the mean using (8) page 299.

$$E(T) = \int_0^\infty G(t)\mathrm{d}t = \int_0^{10} e^{-0.05t}\mathrm{d}t + e^{-0.5} \int_{10}^\infty e^{-0.1(t-10)}\mathrm{d}t = \frac{1}{0.05}\left(1 - e^{-0.5}\right) + \frac{e^{-0.5}}{0.10} = 20 - 10e^{-0.5}$$