IMM - DTU

02405 Probability
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Solution for exercise 4.3.4 in Pitman

The relation between the hazard rate $\lambda(t)$ and the survival function $G(t)$ is given by (7) page 297

$$G(t) = e^{-\int_0^t \lambda(u)\mathrm{d}u}$$

Now inserting $\lambda(u) = \lambda\alpha u^{\alpha-1}$

$$G(t) = e^{-\int_0^t \lambda\alpha u^{\alpha-1}\mathrm{d}u} = e^{-\lambda[u^\alpha]_{u=0}^{u=t}} = e^{-\lambda t^\alpha}$$

Similarly we derive $f(t)$ from $G(t)$ using (5) page 297

$$f(t) = -\frac{\mathrm{d}G(t)}{\mathrm{d}t} = -e^{-\lambda t^\alpha}\left(-\lambda\alpha t^{\alpha-1}\right) = \lambda\alpha t^{\alpha-1}e^{-\lambda t^\alpha}$$

Finally from (6) page 297

$$\lambda(t) = \frac{\lambda\alpha t^{\alpha-1}e^{-\lambda t^\alpha}}{e^{-\lambda t^\alpha}} = \lambda\alpha t^{\alpha-1}$$