

Solution for exercise 4.3.3 in Pitman

Question a) We find $G(t)$ using (7) page 297

$$G(t) = e^{-\int_0^t \frac{a}{b+u} du} = e^{-a[\ln(b+u)]_{u=0}^{u=t}} = e^{-a \ln\left(\frac{b+t}{b}\right)} = \left(1 + \frac{t}{b}\right)^{-a}$$

This is a Pareto distribution. The Pareto distribution is one of the generic distributions with important applications in economics (income distributions), insurance (claim size distribution), geology (distribution for strenght of earth quakes), and telecommunications (duration of internet connections).

Question b) We find $f(t)$ using (5) page 297

$$f(t) = -\frac{dG(t)}{dt} = -\frac{d\left(1 + \frac{t}{b}\right)^{-a}}{dt} = \frac{a}{b} \left(1 + \frac{t}{b}\right)^{-a-1}$$