IMM - DTU

02405 Probability
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BFN/bfn

Solution for exercise 4.2.8 in Pitman

We introduce the events M_i that the transistor is produced on machine i.

Question a) Using the Rule of Average Conditional Probabilities page 41 we get

$$P(X \geq 200) = P(X \geq 200|M_1)P(M_1)+P(X \geq 200|M_2)P(M_2) = e^{-\frac{200}{100}}\frac{4}{12}+e^{-\frac{200}{200}}\frac{8}{12} = 0.2904$$

Question b) Similarly

$$E(X) = 100 \cdot \frac{1}{3} + 200 \cdot \frac{2}{3} = \frac{500}{3}$$

Question c) To find the variance we use the Computational Formula for Variance page 261. We introduce X_i to be the lifetime of a transistor produced by machine i. We use the Computational Formula inversely to get

$$E(X_i^2) = V(X_i) + (E(X_i))^2 = \frac{2}{\lambda_i^2}$$

where $E(X_i) = \frac{1}{\lambda_i}$ is the mean lifetime of a transistor produced on machine i.

$$E(X^2) = E(X_1^2)\frac{1}{3} + E(X_2^2)\frac{2}{3} = 6 \cdot 100^2$$

Finally

$$Var(X) = 6 \cdot 100^2 - \left(\frac{500}{3}\right)^2 = \frac{29}{9} \cdot 100^2$$