

Solution for exercise 4.2.4 in Pitman

Question a) We define T_i as the lifetime of component i . The probability in question is given by the Exponential Survival Function p.279. The mean is 10*hours*, thus $\lambda = 0.1h^{-1}$.

$$P(T_i > 20) = e^{-0.1 \cdot 20} = e^{-2} = 0.1353$$

Question b) The problem is similar to the determination of the *half life* of a radioactive isotope Example 2. p.281-282. We repeat the derivation

$$P(T_i \leq t_{50\%}) = 0.5 \Leftrightarrow e^{-\lambda t_{50\%}} = 0.5 \quad t_{50\%} = \frac{\ln 2}{\lambda} = 6.93$$

Question c) We find the standard deviation directly from page 279

$$SD(T_i) = \frac{1}{\lambda} = 10$$

Question d) The average life time \bar{T} of 100 components is

$$\bar{T} = \frac{1}{100} \sum_{i=1}^{100} T_i$$

We know from page 286 that \bar{T} is Gamma distributed. However, it is more convenient to apply CLT (Central Limit Theorem) p.268 to get

$$P(\bar{T} > 11) = 1 - P(\bar{T} \leq 11) \approx 1 - \Phi\left(\frac{11 - 10}{\frac{10}{\sqrt{100}}}\right) = 1 - \Phi(1) = 0.1587$$

Question e) The sum of the lifetime of two components is Gamma distributed. From p.286 (Right tail probability) we get

$$P(T_1 + T_2 > 22) = e^{-0.1 \cdot 22}(1 + 2.2) = 0.3546$$