

Solution for exercise 4.2.1 in Pitman

We use the knowledge of the half-life to find λ from example 2 page 282,
 $\lambda = \frac{\ln(2)}{1}$

Question a) The probability that an atom survives at least 5 years is given by the survival function page 279. We get with T denoting the life time of an atom

$$P(T > 5) = e^{-5 \ln(2)} = \frac{1}{32}$$

Question b) If we let N_t denote the number of atoms surviving at time t , then the distribution of this random variable will be binomial with the probability found in the previous question. Thus $N_t \in \text{bin}(n, e^{-t \ln(2)})$, where $n = N_0$ is the original number of atoms. The expected value $E(N_t)$ of this binomial distribution is given page 476 or 479 in the distribution summary, such that $E(N_t) = ne^{-t \ln(2)}$. We find $t_{10\%}$ as , using the method on page 282 once more,

$$ne^{-t \ln(2)} = \frac{n}{10} \Rightarrow t_{10\%} = \frac{-\ln(0.1)}{\ln(2)}.$$

Question c) Applying the same method to find the time where the expected number of atoms remaining of 1024 is 10 (t^*), we get

$$1024e^{-t^* \ln(2)} = 10 \Rightarrow t^* = \frac{\ln(1024)}{\ln(2)} = 10$$

Question d) This question can be formulated as $P(N_{t^*} = 0)$. From the binomial distribution of N_{t^*} we get

$$\left(\frac{1023}{1024}\right)^{1024} = 0.3677$$

or

$$\left(\frac{1023}{1024}\right)^{1024} = \left(1 - \frac{1}{1024}\right)^{1024} \simeq e^{-1}$$