

Solution for exercise 4.1.9 in Pitman

We first determine S_4 and $Var(S_4)$. From the distribution summary page 477 we have $E(S_4) = 4\frac{1}{2} = 2$ and due to the independence of the X_i 's we have $Var(S_4) = 4\frac{1}{12} = \frac{1}{3}$. (the result from the variance follows from the result page 249 for a sum of independent random variables and the remarks page 261 which states the validity for continuous distributions). We now have

$$P(S_4 \geq 3) = 1 - \Phi\left(\frac{3-2}{\sqrt{\frac{1}{3}}}\right) = 1 - \Phi(1.73) = 1 - 0.9582 = 0.0418$$