IMM - DTU

02405 Probability
2003-10-13
BFN/bfn

Solution for exercise 4.1.5 in Pitman

Question a)

Question b) We apply the formula on page 263 for a density

$$P(a \leq X \leq b) = \int_a^b f(x)\mathrm{d}x$$

We get

$$P(-1 \leq X \leq 2) = \int_{-1}^{2} \frac{1}{2(1+|x|)^2}\mathrm{d}x = \int_{-1}^{0} \frac{1}{2(1-x)^2}\mathrm{d}x + \int_{0}^{2} \frac{1}{2(1+x)^2}\mathrm{d}x$$

$$= \left[\frac{1}{2(1-x)}\right]_{x=-1}^{x=0} + \left[-\frac{1}{2(1+x)}\right]_{x=0}^{x=2} = \frac{1}{2} - \frac{1}{4} + \frac{1}{2} - \frac{1}{6} = \frac{7}{12}$$

Question c) The distribution is symmetric so $P(|X| > 1) = 2P(X > 1) = 2\left[-\frac{1}{2(1+x)}\right]_{x=1}^{x=\infty} = \frac{1}{2}$.

Question d) No. (the integral $\int_0^\infty x\frac{1}{2(1+x)^2}\mathrm{d}x$ does not exist).