IMM - DTU

02405 Probability
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BFN/bfn

Solution for exercise 4.1.13 in Pitman

Question a) We derive the density of the distribution

$$f(x) = \begin{cases} c(x-0.9) & 0.9 < x \leq 1.0 \\ c(1.1-x) & 1.0 < x < 1.1 \end{cases}$$

We can find c the standard way using $\int_{0.9}^{1.1} f(x)\mathrm{d}x = 1$. However, we can derive the area of the triangle directly as $\frac{1}{2} \cdot 0.02 \cdot c$ such that $c = 10$. Due to the symmetry of $f(x)$ we have $P(X < 0.925) = P(1.075 < X)$.

$$P(\text{rod scrapped}) = 2P(X < 0.925) = 2\int_{0.9}^{0.925} 10(x-0.9)\mathrm{d}x = 20\left[\frac{1}{2}x^2 - 0.9x\right]_{x=0.9}^{x=0.925} = 0.0625$$

Question b) We define the random variable Y as the length of an item which has passed the quality inspection. The probability

$$P(0.95 < Y < 1.05) = \frac{P(0.95 < X < 1.05)}{P(0.925 < X < 1.075)} = \frac{0.75}{0.9375} = 0.8$$

The number of acceptable items A out of c are binomially distributed. We determine c such that

$$P(A \geq 100) \geq 0.95$$

We now use the normal approximation to get

$$1 - \Phi\left(\frac{100 - 0.5 - 0.8 \cdot c}{0.4\sqrt{c}}\right) \geq 0.95$$

$$\frac{100 - 0.5 - 0.8 \cdot c}{0.4\sqrt{c}} \leq -1.645$$

and we find $c \geq 134$.