Solution for exercise 3.5.9 in Pitman

Question a)

$$P(X=1,Y=2) = P(X=1)P(Y=2) = \frac{1^1}{1!}e^{-1}\frac{2^2}{2!}e^{-2} = 2e^{-3}$$

Question b)

$$P\left(\frac{X+Y}{2} \geq 1\right) = P(X+Y \geq 2) = 1 - P(X+Y \leq 1)$$

$$= 1 - (P(X+Y=0) + P(X+Y=1)) = 1 - (1+3)e^{-3} = 0.80,$$

where we use a) to find $P(X+Y=0)$ and $P(X+Y=1)$.

Question c)

$$P\left(X=1|\frac{X+Y}{2}=2\right) = P\left(X=1|X+Y=4\right) = \frac{P(X=1,X+Y=4)}{P(X+Y=4)}$$

$$= \frac{e^{-1}\frac{2^3}{3!}e^{-2}}{\frac{3^4}{4!}e^{-3}} = \left(\begin{array}{c} 4 \\ 1 \end{array}\right)\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^3 = 0.395$$

the conditional probability is given by the Binomial distribution. This is a general result.