IMM - DTU

02405 Probability
2003-10-5
BFN/bfn

Solution for exercise 3.5.18 in Pitman

Question a) The variable X_1 is the sum of a thinned Poisson variable (X_0) and a Poisson distributed random variable (the immigration). The two contributions are independent, thus X_1 is Poisson distributed. The same argument is true for any n and we have proved that X_n is Poisson distributed by induction. Ee denote the parameter of the n'th distribution by λ_n. We have the following recursion:

$$\lambda_n == p\lambda_{n-1} + \mu$$

with $\lambda_0 = \mu$ such that

$$\lambda_1 = (1+p)\mu$$

and more generally

$$\lambda_n = \sum_{i=0}^{n} p^i \mu = \mu \frac{1-p^{n+1}}{1-p}$$

Question b) As $n \to \infty$ we get $\lambda_n \to \frac{\mu}{1-p}$. This value is also a fixpoint of

$$\lambda_n == p\lambda_{n-1} + \mu$$