Solution for exercise 3.5.16 in Pitman

We assume that the chocolate chips and mashmallows are randomly scattered in the dough.

Question a) The number of chocoloate chips in one cubic inch is Poisson distributed with parameter 2 according to our assumptions. The number of chocolate chips in thre cubic inches is thus Poisson distributed with parameter 6. Let X denote the number of chocolate chops in a three cubic inch cookie.

$$P(X \leq 4) = e^{-6} \left(1 + 6 + \frac{36}{2} + \frac{36 \cdot 6}{6} + \frac{216 \cdot 6}{4 \cdot 6} \right) = 115 \cdot e^{-6} = 0.285$$

Question b) We have three Poisson variates X_i : total number of chocolate chips and marshmallows in cookie i. According to our assumptions, X_1 follows a Poisson distribution with parameter 6, while X_2 and X_3 follow a Poisson distribution with parameter 9. The complementary event is the event that we get two or three cookies without chocoloate chips and marshmallows.

$$P(X_1 = 0, X_2 = 0, X_3 = 0) + P(X_1 > 1, X_2 = 0, X_3 = 0)$$

$$+P(X_1 = 0, X_2 > 1, X_3 = 0) + P(X_1 = 0, X_2 = 0, X_3 > 1)$$
$$= e^{-6}e^{-9}e^{-9} + (1 - e^{-6})e^{-9}e^{-9} + e^{-6}(1 - e^{-9})e^{-9} + e^{-6}e^{-9}(1 - e^{-9}) \tilde{=} 0$$

we are almost certain that we will get at most one cookie without goodies.