

**Solution for exercise 3.5.10 in Pitman**

**Question a)**  $X$  is Poisson distributed with parameter  $\lambda$ . Using page 175, linear functions of  $X$ ,

$$E(3X + 5) = 3E(X) + 5$$

and the mean of a Poisson distributed random variable page 223

$$E(3X + 5) = 3E(X) + 5 = 3\lambda + 5$$

**Question b)** Using linear functions of  $X$  page 188, - here called Scaling and Shifting, and the variance of a Poisson distributed random variable page 223.

$$V(3X + 5) = 9V(X) = 9\lambda$$

**Question c)** We use the definition of the expectation of a function of a random variable p.175

$$\begin{aligned} \sum_{i=0}^{\infty} \frac{1}{i+1} \frac{\lambda^i}{i!} e^{-\lambda} &= e^{-\lambda} \sum_{i=0}^{\infty} \frac{\lambda^i}{(i+1)!} \\ &= \frac{e^{-\lambda}}{\lambda} \sum_{i=0}^{\infty} \frac{\lambda^{i+1}}{(i+1)!} \end{aligned}$$

Now

$$\sum_{i=0}^{\infty} \frac{a^i}{i!} = e^a$$

such that

$$\frac{e^{-\lambda}}{\lambda} \sum_{i=0}^{\infty} \frac{\lambda^{i+1}}{(i+1)!} = \frac{1}{\lambda} (1 - e^{-\lambda})$$