Solution for exercise 3.4.9 in Pitman

We define the random variable N as the number of throws to get heads. The pay back value is N^2, the expected win from the game can be expressed as

$$E(N^2 - 10) = E(N^2) - 10$$

using the rule for the expectation of a linear function of a random variable p. 175 b. We could derive $E(N^2)$ from the general rule for expectation of a function of a random variable p. 175 t. However, it is more convenient to use the fact the N follows a Geometric distribution and use the Computational Formula for the Variance p. 186.

$$E(N^2) = Var(N) + (E(N))^2 = \frac{1-p}{p^2} + \left(\frac{1}{p}\right)^2 = 2 + 4 = 6$$

The values for $Var(N)$ and $E(N)$ can be found p. 476 in the distribution summary.