IMM - DTU

02405 Probability
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BFN/bfn

# Solution for exercise 3.1.24 in Pitman

**Question a)** We define $P(X \text{ even}) = P(Y \text{ even}) = p$, and introduce the random variable $W = X + Y$. The probability $p_w$ of the event that $W$ is even is

$$p_w = p^2 + (1-p)(1-p) = 2p^2 + 1 - 2p = (1-p)^2 + p^2$$

with minimum $\frac{1}{2}$ for $p = \frac{1}{2}$.

**Question b)** We introduce $p_0 = P(X \mod 3 = 0), p_1 = P(X \mod 3 = 1), p_2 = P(X \mod 3 = 2)$. The probability in question is

$$p_0^3 + p_1^3 + p_2^3 + 3p_0p_1p_2$$

which after some manipulations can be written as

$$1 - (p_0p_1 + p_0p_2 + p_1p_2 - 3p_0p_1p_2)$$

The expressions can be maximized/minimized using standard methods, I haven't found a more elegant solution than that.