

Solution for exercise 3.1.16 in Pitman

Question a) Using the law of averaged conditional probabilities we get

$$P(X+Y = n) = \sum_{i=0}^n P(X = i)P(X+Y = n|X = i) = \sum_{i=0}^n P(X = i)P(Y = n-i)$$

where the last equality is due to the independence of X and Y .

Question b) The marginal distribution of X and Y is

$$\begin{aligned} P(X = 2) &= \frac{1}{36}, & P(X = 3) &= \frac{1}{18}, & P(X = 4) &= \frac{1}{12} \\ P(X = 5) &= \frac{1}{9}, & P(X = 6) &= \frac{5}{36}, & P(X = 7) &= \frac{1}{6} \end{aligned}$$

We get

$$P(X + Y = 8) = 2 \left(\frac{1}{36} \cdot \frac{5}{36} + \frac{1}{18} \cdot \frac{1}{9} \right) + \frac{1}{12} \cdot \frac{1}{12} = \frac{35}{16 \cdot 81}$$