IMM - DTU

02405 Probability
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BFN/bfn

# Solution for exercise 3.1.14 in Pitman

**Question a)** We define the events $Gg$ as the the events that team $A$ wins in $g$ games. The probabilities $P(Gg)$ can be found by thinking of the game series as a sequence of Bernoulli experiments. The event $Gg$ is the event that the fourth succes (win by team $A$) occurs at game $g$. These probabiliites are given by the negative binomial distribution (page 213 or page 482). Using the notation of the distribution summary page 482, we identify $r = 4$, $n = g - 4$ (i.e. counting only the games that team $A$ loses). We get

$$P(Gg) = \binom{g-1}{4-1} p^4 q^{g-4} \qquad g = 4, 5, 6, 7$$

**Question b)**

$$p^4 \sum_{g=4}^{7} \binom{g-1}{3} q^{g-4}$$

**Question c)** The easiest way is first answering question d) then using $1 - binocdf(3, 7, 2/3)$ in MATLAB.

$$0.8267$$

**Question d)** Imagine that all games are played etc. From the binomial formula

$$p^7 + 7p^6q + 21p^5q^2 + 35p^4q^3 = p^7 + p^6q + 6p^6q + 6p^5q^2 + 15p^5q^2 + 35p^4q^3$$

$$= p^6 + 6p^5q + 15p^4q^2 + 20p^4q^3 = p^6 + p^5q + 5p^5q + 15p^4q^2 + 20p^4q^3$$

etc.

**Question e)**

$$P(G = 4) = p^4 + q^4 \qquad P(G = 5) = 4pq(p^3 + q^3)$$

$$P(G = 6) = 10p^2q^2(p^2 + q^2) \qquad P(G = 7) = 20p^3q^3(p + q)$$

Independence for $p = q = \frac{1}{2}$