

Solution for exercise 2.5.9 in Pitman

Question a) The probability that the second sample is drawn is the probability that the first sample contains exactly one bad item, which occurs with probability

$$p_1 = \frac{\binom{10}{1} \binom{40}{4}}{\binom{50}{5}}$$

(the hypergeometric distribution page 125). The probability that the second sample contains more than one bad item is calculated via the probability of the complementary event, i.e. that the second sample contains one or two bad items, which is

$$p_2 = \frac{\binom{9}{0} \binom{36}{10}}{\binom{45}{10}} + \frac{\binom{9}{1} \binom{36}{9}}{\binom{45}{10}}$$

The answer to the question is the product of these two probabilities $p_1(1 - p_2) = 0.2804$.

Question b) The lot is accepted if we have no bad items in the first sample or the event described under a)

$$\frac{\binom{10}{0} \binom{40}{5}}{\binom{50}{5}} + \frac{\binom{10}{1} \binom{40}{4}}{\binom{50}{5}} \left(\frac{\binom{9}{0} \binom{36}{10}}{\binom{45}{10}} + \frac{\binom{9}{1} \binom{36}{9}}{\binom{45}{10}} \right) = 0.4595$$