IMM - DTU

02405 Probability
2005-2-20
BFN/bfn

Solution for exercise 2.5.5 in Pitman

The probability in question is given by the Binomial distribution evaluated with the Normal approximation (boxed result page 99). Let A_i define the event that i voters in the sample prefer A. Then $P(A_i)$ is given by the $Bin(n, 0.55)$ distribution. We want to determine n such that $P\left(\cup_{i>\frac{n}{2}} A_i\right) \geq 0.99 \Leftrightarrow P\left(\cup_{i\leq\frac{n}{2}} A_i\right) \leq 0.01$. Expressed differently $P\left(0 \leq \text{Number preferring } B \leq \frac{n}{2}\right)$.

$$\Phi\left(\frac{\frac{n}{2}+\frac{1}{2}-0.55\cdot n}{\sqrt{n\cdot 0.55\cdot 0.45}}\right) \leq 0.99$$

Thus

$$\frac{\frac{n}{2}+\frac{1}{2}-0.55\cdot n}{\sqrt{n\cdot 0.55\cdot 0.45}} \leq -2.33 \Rightarrow n > 557 \quad .$$

Pitman gets 537 ignoring the continuity approximation.