IMM - DTU

02405 Probability
2003-9-19
BFN/bfn

Solution for exercise 2.4.8 in Pitman

The Poisson probabilities $P_\mu(k)$ are

$$P_\mu(k) = \frac{\mu^k}{k!} e^- \mu$$

We use odds ratio for the probabilities

$$\frac{P(k+1)}{P(k)} = \frac{\frac{\mu^{k+1}}{(k+1)!} e^- \mu}{\frac{\mu^k}{k!} e^- \mu} = \frac{\mu}{k+1}$$

The ratio is strictly decreasing in k. For $\mu < 1$ maximum will be $P_\mu(0)$, otherwise the probabilities will increase for all k such that $\mu > k$, and decrease whenever $\mu < k$. For non-integer μ the maximum of $P_\mu(k)$ (the mode of the distribution) is obtained for the largest $k < \mu$. For μ intger the value of $P_\mu(\mu) = P_\mu(\mu + 1)$.