IMM - DTU

02405 Probability
2004-2-10
BFN/bfn

Solution for exercise 2.1.4 in Pitman

We denote the event that there are 3 sixes in 8 rolls by A, the event that there are 2 sixes in the first 5 rolls by B. The probability in question is $P(B|A)$. Using the general formula for conditional probabilities page 36

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

The probability $P(B \cap A) = P(A|B)P(B)$ by the multiplication rule, thus as a speical case of Bayes Rule page 49 we get

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{P(A|B)P(B)}{P(A)}$$

Now the probability of $P(A)$ is given by the binomial distribution page 81, as is $P(B)$ and $P(A|B)$ (the latter is the probability of getting 1 six in 3 rolls). Finally

$$P(B|A) = \frac{P(2 \text{ sixes in 5 rolls})P(1 \text{ six in 3 rolls})}{P(3 \text{ sixes in 8 rolls})} = \frac{\binom{5}{2}\frac{5^3}{6^5}\binom{3}{1}\frac{5^2}{6^3}}{\binom{5}{2}\frac{5^5}{6^8}} = \frac{\binom{5}{2}\binom{3}{1}}{\binom{8}{3}}$$

a hypergeometric probability. The result generalizes. If we have x successes in n trials then the probability of having $y \leq x$ successes in $m \leq n$ trials is given by

$$\frac{\binom{m}{y}\binom{n-m}{x-y}}{\binom{n}{x}}$$

The probabilities do not depend on p.