Solution for exercise 1.6.8 in Pitman

question a) The events B_{ij} occur with probability

$$P(B_{ij}) = \frac{1}{365}$$

It is immediately clear that

$$P(B_{12} \cap B_{23}) = \frac{1}{365^2} = P(B_{12})P(B_{23}).$$

implying independence. The following is a formal and lengthy argument. Define

$A_{i,j}$ as the the event that the i'th person is born the j'th day of the year.

We have $P(A_{i,j}) = \frac{1}{365}$ and that A_{1_i}, $A_{2,j}$, and $A_{3,k}$ are independent. The event B_{ij} can be expressed by

$$B_{ij} = \cup_{k=1}^{365} (A_{i,k} \cap A_{j,k})$$

such that $P(B_{ij}) = \frac{1}{365}$ by the independence of $A_{i,k}$ and $A_{j,k}$. The event $B_{12} \cap B_{23}$ can be expressed by

$$B_{12} \cap B_{23} = \cup_{k=1}^{365} (A_{1,k} \cap A_{2,k} \cap A_{3,k})$$

and by the independence of the A's we get $P(B_{12} \cap B_{23}) = \frac{1}{365^2}$

question b) The probability

$$P(B_{13}|B_{12} \cap B_{23}) = 1 \neq P(B_{13})$$