IMM - DTU

02405 Probability
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BFN/bfn

Solution for exercise 1.6.5 in Pitman

Question a) We will calculate the complementary probability, the no student has the same birthday and do this sequentially. The probability that the first student has a different birthday is $\frac{364}{365}$, the same is true for all the remaining $n-2$ students. The probability in question is

$$P(\text{All other } n-1 \text{ students has a different birthday than no.1}) = 1 - \left(\frac{364}{365}\right)^{n-1}$$

Question b)

$$1 - \left(\frac{364}{365}\right)^{n-1} \geq \frac{1}{2} \Leftrightarrow n \geq \frac{\ln(2)}{\ln(365) - \ln(364)} + 1 = 253.7$$

Question c) In the birthday problem we only ask for two arbitrary birthdays to be the same, while the question in this exercise is that at least one out of $n-1$ has a certain birthday.