Solution for exercise 1.5.5 in Pitman

Define the events

H A randomly selected person is healthy

D A randomly selected person is diagnosed with the disease

Question a) From the text we have the following quantities

$$P(H) = 0.99 \qquad P(D|H) = 0.05 \qquad P(D|H^c) = 0.8$$

and from the law of averaged conditional probabilities we get

$$P(D) = P(H)P(D|H) + P(H^c)P(D|H^c) = 0.99 \cdot 0.05 + 0.01 \cdot 0.8 = 0.0575$$

Question b) The proability in question

$$P(H^c \cap D^c) = P(H^c)P(D^c|H^c) = 0.01 * 0.2 = 0.002$$

using the multiplication (chain) rule

Question c) The proability in question

$$P(H \cap D^c) = P(H)P(D^c|H) = 0.99 * 0.95 = 0.9405$$

using the multiplication (chain) rule

Question d) The probability in question is $P(H^c|D)$. We use Bayes rule to "interchange" the conditioning

$$P(H^c|D) = \frac{P(D|H^c)P(H^c)}{P(D|H^c)P(H^c) + P(D|H)P(H)} = 0.8 \cdot 0.010.008 + 0.05 \cdot 0.99 = 0.139$$

Question e) The probabilities are estimated as the percentage of a large group of people, which is indeed the frequency interpretation.