IMM - DTU

02405 Probability
2013-9-5
BFN/bfn

Solution for exercise 1.4.11 in Pitman

We define the events BB, BG, GB, and GG as e.g. for BG the event that the firstborn is a boy and the secondborn is a girl. The events $B.$ and $.B$ are the events the firstbon respectively secondborn is a boy. Finally we define the event I to be the event that the children are identical twins.

Question a)

$$P(BB) = P(I)P(BB|I) + P(I^c)P(BB|I^c) = p \cdot 1 + (1-p) * \frac{1}{4} = \frac{1+p}{4}$$

Question b)

$$P(BG) = P(I)P(BG|I) + P(I^c)P(BG|I^c) = p \cdot 0 + (1-p) * \frac{1}{4} = \frac{1-p}{4}$$

Question c)

$$P(.G|B.) = P(BG \cup GG|BB \cup BG) = \frac{P(BG)}{P(BB \cup BG)} = \frac{1-p}{2}$$

Question d)

$$P(.G|G.) = \frac{1+p}{2}$$

You should check the results by inserting $p = 0$ and $p = 1$.