

Solution for exercise 1.3.9 in Pitman

Question a)

$$P(F \cup G) = P(F) + P(G) - P(F \cap G) = 0.7 + 0.6 - 0.4 = 0.9$$

using exclusion-inclusion.

Question b)

$$\begin{aligned} P(F \cup G \cup H) &= P(F) + P(G) + P(H) - P(F \cap G) - P(F \cap H) - P(G \cap H) + P(F \cap G \cap H) \\ &= 0.7 + 0.6 + 0.5 - 0.4 - 0.3 - 0.2 + 0.1 = 1.0 \end{aligned}$$

using the general version of exclusion-inclusion (see exercise 1.3.11 and 1.3.12).

Question c)

$$\begin{aligned} P(F^c \cap G^c \cap H) &= P((F \cup G)^c \cap H) \\ P(H) &= P((F \cup G)^c \cap H) + P((F \cup G) \cap H) \end{aligned}$$

The latter probability is

$$\begin{aligned} P((F \cup G) \cap H) &= P((F \cap H) \cup (G \cap H)) = P(F \cap H) + P(G \cap H) - P(F \cap G \cap H) \\ &= 0.3 + 0.2 - 0.1 = 0.4 \end{aligned}$$

such that

$$P(F^c \cap G^c \cap H) = 0.5 - 0.4 = 0.1$$