IMM - DTU

02405 Probability
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BFN/bfn

Solution for exercise 5.2.4 in Pitman

We can rewrite the density

$$f(x,y) = 2e^{-2x}3e^{-3y}$$

to see that X and Y are independent expontially distributed random variables which basically solves a)-c). Alternatively:

Question a) The area B page 349 is defined by the rectangle $0 < u < x, 0 < v < y$.

$$P(X \leq x, Y \leq y) = \int_0^x \int_0^y f(u,v) \mathrm{d}v \mathrm{d}u = \int_0^x \int_0^y 2e^{-2u}3e^{-3v} \mathrm{d}v \mathrm{d}u$$

$$= \int_0^x 2e^{-2u}\left(1 - e^{-3y}\right) \mathrm{d}u = \left(1 - e^{-2x}\right)\left(1 - e^{-3y}\right)$$

Question b)

$$f_X(x) = 2e^{-2x}$$

Question c)

$$f_Y(y) = 3e^{-3y}$$

Question d) The variables X and Y are independent since

$$f(x,y) = f_X(x) f_Y(y)$$

for all (x,y).