Solution for exercise 5.2.1 in Pitman

Question a) A nice drawing The area of the figure (shaded area) is 1.

$$f(x,y) = 1, \qquad 0 < |y| < x$$

Question b) We find the marginal distribution of X by integrating over y for fixed x (page 349)

$$f_X(x) = \int_{-x}^{x} 1 \cdot \mathrm{d}y = 2x, \qquad 0 < x < 1$$

Similarly for positive y

$$f_Y(y) = \int_{y}^{1} 1 \cdot \mathrm{d}x = 1 - y, \qquad 0 < y < 1$$

and for negative y

$$f_Y(y) = \int_{-y}^{1} 1 \cdot \mathrm{d}x = 1 + y, \qquad -1 < y < 0$$

leading to a general expression for $f_Y(y)$

$$f_Y(y) = 1 - |y|, \qquad -1 < y < 1$$

Question c) No. (e.g. $P\left(|Y| > \frac{1}{2} | X < \frac{1}{2}\right) = 0 \neq \frac{1}{4} = P\left(|Y| > \frac{1}{2}\right)$.

Question d) From the definiton of $E(X)$ page 261 (page 332)

$$E(x) = \int_{0}^{1} x \cdot 2x \mathrm{d}x = \frac{2}{3}$$

The distribution of Y is symmetric around 0 so $E(Y) = 0$.