IMM - DTU

02405 Probability
2006-11-3
BFN/bfn

Solution for exercise 4.6.1 in Pitman

We introduce the random variables $X_i; i = 1, 2, 3, 4$ for the arrival time of the i'th person. For convenience X_i will be the deviation from 12 noon measured in minutes.

Question a) Since X_i are continuous random variables the question can be stated as

$$P(\min_i X_i < -10) = P(\min_i X_i \leq -10)$$

From the result page 317 and the normality of the X_i's we get

$$P(\min_i X_i \leq -10) = 1 - \left(1 - \Phi\left(-\frac{10}{5}\right)\right)^4 = 1 - 0.9772^4 = 0.088$$

(compared with the probability 0.0228 that a specific person will arrive before 11.50)

Question b) This question can be stated as

$$P(\max_i (X_i) > 15) = 1 - P(\max_i (X_i) \leq 15) = 1 - \Phi\left(\frac{15}{5}\right)^4 = 1 - 0.9987^4 = 0.0052$$

from the result regarding the distribution of the maximum of indpendent random variables page 316.

Question c) The question regards the second order distribution i.e. the distribution of $X_{(2)}$ where $X_{(1)} \leq X_{(2)} \leq X_{(3)} \leq X_{(4)}$. The expression for this density is stated page 326. With $x = 0$, $\mathrm{d}x = 2 \cdot \frac{1}{6}$, and $f(x) = \frac{1}{5\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x}{5}\right)^2}$ (page 267) we get

$$P\left(-\frac{1}{6} \leq X_{(2)} \leq \frac{1}{6}\right) \simeq f_{(2)}(0)\frac{2}{6} = 4 \binom{3}{1} \frac{1}{5\sqrt{2\pi}} \frac{2}{6} \frac{1}{2} \left(\frac{1}{2}\right)^2 = 0.0399$$

(we have used $F(0) = \Phi\left(\frac{0-0}{5}\right) = \frac{1}{2}$)