IMM - DTU

02405 Probability
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BFN/bfn

Solution for exercise 4.1.12 in Pitman

Question a) First we determine the total area of the figure, which is 8. The area of the triangle with x-coordinate less than or equal to x_0 is $\frac{1}{2}(x+2)\cdot 2(x+2) = (x+2)^2$ for $x \leq 0$ and $8 - \frac{1}{2}(2-x)2(2-x) = 4 + 4x - x^2$ for $0 < x \leq 2$. We find

$$f(x) = \begin{cases} \frac{x+2}{4} & -2 \leq x \leq 0 \\ \frac{2-x}{4} & 0 < x \leq 2 \end{cases}$$

Question b) We can write the density as

$$f(x) = \begin{cases} c(2+x) & -2 \leq x < 0 \\ 2c(1-x) & 0 \leq x \leq 1 \end{cases}$$

From integration (or by considering the area of the figure) we find $c = \frac{1}{3}$.

Question c) The four lines defining the square are: $y = 2x-3, y = 2-\frac{1}{2}x, y = 2x+2$ and $y = -\frac{1}{2}x-\frac{1}{2}$. The area of the square is 5. Now considering $P(x < X < x+\mathrm{d}x)$ for $-1 < x < 0$. The triangle defined by the vertical line through $x, y = 2x+2$ and $y = -\frac{1}{2}x - \frac{1}{2}$ has area $\frac{1}{2}x(2x+2 - -\frac{1}{2}x - \frac{1}{2} = \frac{1}{4}(x+1)^2$. We find the area of the triangle defined by the vertical line through $x + \mathrm{d}x, y = 2x+2$ and $y = -\frac{1}{2}x - \frac{1}{2}$ to $\frac{1}{4}(x + \mathrm{d}x + 1)^2$ and derive $f(x)\mathrm{d}x = \frac{1}{2}(x+1)$. By using similar arguments for the intervals $(0,1)$ and $(1,2)$ we get

$$f(x) = \begin{cases} \frac{1}{2}(1+x) & -1 \leq x < 0 \\ \frac{1}{2} & 0 \leq x < 1 \\ \frac{1}{2}(2-x) & 1 \leq x \leq 2 \end{cases}$$