Solution for exercise 4.1.2 in Pitman

Question a) The integral of $f()$ should be one for $f()$ to be a density (p.263).

$$\int_1^\infty \frac{c}{x^4}\mathrm{d}x = [-\frac{c}{3}\frac{1}{x^3}]_{x=1}^{x=\infty} = \frac{c}{3}$$

We conclude $c = 3$.

Question b) Using the definition of $E(g(X))$ page 263, we find the mean of X to be

$$\int_1^\infty xf(x)\mathrm{d}x = \int_1^\infty x\frac{3}{x^4}\mathrm{d}x = \frac{3}{2}$$

Question c) The Computational Formula for Variance is still valid (page 261). We get

$$E(X^2) = \int_1^\infty x^2 f(x)\mathrm{d}x = \int_1^\infty x^2 \frac{3}{x^4}\mathrm{d}x = 3, \qquad Var(X) = 3 - \left(\frac{3}{2}\right)^2 = \frac{3}{4}$$