

Solution for exercise 3.4.1 in Pitman

Question a) X the number of heads in 9 tosses is binomially distributed, thus

$$P(X = 5) = \binom{9}{5} p^5 (1-p)^4$$

Question b) Y the number of tosses for the first head is geometrically distributed, thus

$$P(Y = 7) = (1-p)^6 p$$

Question c) Z the number of tosses to get 5 heads follows a negative binomial distribution

$$P(Z = 12) = \binom{11}{4} (1-p)^7 p^5$$

Question d) X_1 the number of heads in the first 8 tosses and X_2 the number of heads in the next 5 tosses are independent. We get

$$\sum_{i=0}^5 \binom{8}{i} p^i (1-p)^{8-i} \binom{5}{i} p^i (1-p)^{5-i} = \sum_{i=0}^5 \binom{8}{i} \binom{5}{i} p^{2i} (1-p)^{13-2i}$$