

Solution for exercise 3.3.4 in Pitman

The computational formula for the variance page 186 is quite useful (important). This exercise is solved by applying it twice. First we use it once to get:

$$\text{Var}(X_1X_2) = E((X_1X_2)^2) - (E(X_1X_2))^2$$

Now by the independence of X_1 and X_2

$$E((X_1X_2)^2) - (E(X_1X_2))^2 = E(X_1^2X_2^2) - (E(X_1)E(X_2))^2 = E(X_1^2)E(X_2^2) - (E(X_1)E(X_2))^2$$

using the multiplication rule for Expectation page.177 valid for independent random variables. We have also used the fact that if X_1 and X_2 are independent then $f(X_1)$ and $g(X_2)$ are independent too, for arbitrary functions $f()$ and $g()$. We now use the computational formula for the variance once more to get

$$\text{Var}(X_1X_2) = (\text{Var}(X_1) + (E(X_1))^2)(\text{Var}(X_2) + (E(X_2))^2) - (E(X_1)E(X_2))^2$$

Now inserting the symbols of the exercise we get

$$\text{Var}(X_1X_2) = \sigma_1^2\sigma_2^2 + \mu_1^2\sigma_2^2 + \mu_2^2\sigma_1^2$$